RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2016-19]

B.A./B.Sc. FIRST SEMESTER (July – December) 2016

Mid-Semester Examination, September 2016

Date : 10/09/2016

MATHEMATICS (Honours)

Time : 11 am – 1 pm

Paper : I

Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

(*Answer any four questions*) [4×3]

1. For any three nonempty sets A, B and C prove that $A\times(B-C)=(A\times B)-(A\times C)$.

2. Let S be a finite set with n elements. Prove that the number of binary relations on S that are both reflexive and symmetric is $2^{\frac{n(n-1)}{2}}$.

3. Let us consider the map $f:A\to B$. Show that f is left invertible if and only if f is injective.

4. Prove that a semigroup $(S,*)$ is a group if and only if left identity exists and left inverse for every element exists in S.

5. Let H and K be two subgroups of a group G. Show that HK is a subgroup of G if and only if HK = KH.

6. Prove that every partition of a set A yields an equivalence relation on A.

Group – B

7. **Answer any three questions :** [3×5]

   a) If $x, y\in\mathbb{R}$ with y > 0, then prove that there exist $n\in\mathbb{N}$ such that ny > x. [5]

   b) Define an open set in $\mathbb{R}$. Show that the union of an arbitrary collection of open sets in $\mathbb{R}$ is an open set in $\mathbb{R}$. [1+4]

   c) Define limit point of a set in $\mathbb{R}$. Prove that the derived set of a set in $\mathbb{R}$ is closed. [1+4]

   d) Define convergence of a sequence.
   If $\lim u_n = u$, $\lim v_n = v$ then show that $\lim(u_n v_n) = uv$. [1+4]

   e) Prove that the sequence $\{u_n\}_n$ defined by $u_1=\sqrt{2}$ and $u_{n+1}=\sqrt{2u_n}$ $\forall n\in\mathbb{N}$ converges to 2. [5]

8. **Answer any two questions :** [2×5]

   a) Show that the equation of the line joining the feet of perpendiculars from the point (d, 0) on the lines $ax^2+2hxy+by^2=0$ is $(a-b)x+2hy+bd=0$. [5]

   b) Find the chord of contact of the conic $\frac{\ell}{r}=1+e\cos\theta$ joining the points of contact of the tangents drawn from the external point (r',θ'). [5]

   c) Prove that the conics $\frac{\ell_1}{r}=1-e_1\cos\theta$ and $\frac{\ell_2}{r}=1-e_2\cos(\theta-\alpha)$ will touch one another if $\ell_1^2(1-e_2^2)+\ell_2^2(1-e_1^2)=2\ell_1\ell_2(1-e_1e_2\cos\alpha)$. [5]

Group – C

9. Answer **any two** questions : [2×4]

a) Prove that $2\frac{dy}{dx}\frac{d^3y}{dx^3}=3\left(\frac{d^2y}{dx^2}\right)^2$ if $y=\frac{\ell x+m}{px+q}$ where ℓ,m, p, q are arbitrary constants and if $\ell+q=0$ then prove that $(y-x)\frac{d^2y}{dx^2}=2\left(1+\frac{dy}{dx}\right)\frac{dy}{dx}$.

b) Solve : $(xy+2x^2y^2)ydx+(xy-x^2y^2)xdy=0$.

c) Solve the equation by the method of variation of parameters $\frac{dx}{dt}+\frac{a}{t}x=\frac{b}{t^n}$ where n is any positive real member, a and b are constants.

10. Answer **any one** question : [1×5]

a) Prove that the necessary and sufficient condition that $Mdx+Ndy=0$ will be exact when $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$.

b) Solve : $y-px=\phi(x^2+y^2)\sqrt{1-p^2}$, where $p=\frac{dy}{dx}$ and ϕ is any function with positive value.

——————— × ———————